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BACK ANALYSIS OF ELECTRO-LEVEL READINGS INSTALLED IN THE SLAB OF THE UHE MACHADINHO DAM

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ABSTRACT

In recent years, the construction of concrete faced rockfill dams has generated discussions concerning the design of concrete slabs in the upstream slope. This stage of the design is usually carried out by means of empirical expressions, which determine the thickness of the slab and the steel reinforcement. Nowadays, optimization of the material layers, which constitute the rockfill, and the increase in the dam height have, however, demanded more detailed studies, specially related to structural behavior.

In this paper, using the data from electro-levels installed in the slab of UHE Machadinho, back analysis is applied to identify Young's modulus of the material used in a particular region of the rockfill dam. This study is rooted in previous works by the authors, in which these data were employed to identify the loading parameters of the reaction of the rockfill dam on the concrete face.

INTRODUCTION

With the technological advance in the construction of concrete faced rockfill dams (CFRD), one notices their increase in height and the constant increase in usage of materials from excavations of the rockfill dam. At the same time, the monitoring of these structures has supplied information, which provides an outstanding progress in the interpretation of the behavior of such dams, in the evaluation of their performance and in the re-evaluation of their design criteria.

The study of the concrete face of the dam, responsible for the impermeability of the upstream face, is particularly important. An empirical method has predominated in the analysis of concrete slabs using expressions developed to determine their thicknesses, the steel reinforcement distribution and rates. However, structural analysis resources and information gathered by monitoring have provided studies on the concrete slab behavior, which may change former procedures, allowing the verification of the dimensions of the concrete slab and the steel reinforcement rates, and therefore the improvement of design criteria.

This paper presents studies of the concrete slab 24 of the concrete faced rockfill UHE Machadinho Dam, located at the Pelotas River, in the south of Brazil. The dam stands at a maximum height of 125m, with a crest length of approximately 673m. Electro-levels were mounted on the dam's central slab 24 and on two other side slabs (14 and 36) to monitor slab deformation. Figure 1 presents the electro-level positions on slab 24.

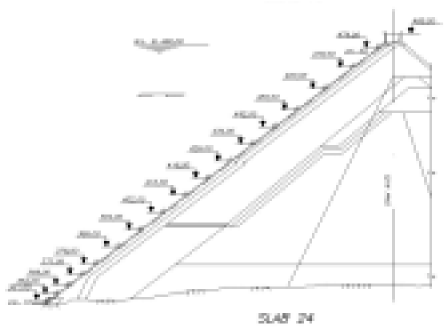

Figure 1. Electro-level positions on slab 24

The dam was studied by the authors in [1], [2] and [3]. In those papers, a one-dimensional model was considered to represent the concrete slab and measured rotations were utilized to determine, by back analysis, the reaction of the rockfill dam on the concrete face. Those results were the basis to obtain displacements, rotations and a bending moment diagrams for the concrete slab.

In this paper, a two-dimensional mathematical model is adopted to represent the structure and the same measured rotations are used. Back analysis is applied to identify the Young's modulus of the material of one of the embankment regions close to the concrete face. Next, one performs a structural analysis with the Young's modulus obtained. The results – displacements, rotations and bending moments in concrete slab – obtained using the two-dimensional model and their comparison to the results obtained with the one-dimensional model are the main objectives of this paper. One notices that from the bending moment diagram it is possible to perform a series of studies that permit a better evaluation of the concrete slab performance, so as to verify recommendations of empirical criteria and to re-evaluate existing design criteria.

GENERAL FORMULATION OF THE BACK ANALYSIS PROBLEMS

In order to represent the reality of a given physical phenomenon, systems are indispensable. They are usually defined as a set of elements whose interaction is governed by a certain law.

The most convenient way to represent a system is by the definition of a physical or mathematical model that is able to simulate it well as to the main aspects of its behavior under certain conditions.

To define a system is to establish the relationships among the main parts of the system: input signals, system properties and output signals. Input signals are the actions exerted by the environment on the system; output signals are answers that specify system behavior; and system properties are characterized by laws defining the relationship between both signals.

In system analysis, input signals and system properties are known and output signals are to be determined. In system identification, which comprises back analysis problems, the output signals are combined with system properties to determine input signals, or combined with input signals to determine system properties.

In structural engineering, models are usually well-known and therefore the identification problem is reduced to finding model parameters that lead to the best relationship between measured quantities and values calculated by the model, be they related to structure geometry, material properties or actions.

Therefore, the identification of parameters is equivalent to the mathematical problem of minimizing a properly defined function of those parameters, which involves the difference between measured values and calculated values.

The definition of the function to be minimized, labeled objective function $(J(\mathbf{p}))$, is a consequence of identification criterion.

The simplest criterion used in this paper, is the minimum square criterion: it requires no previous knowledge of measurement deviations and of the parameters to be determined and it relies on the minimization of the function

$$J(\mathbf{p}) = [\mathbf{u}^* - \mathbf{u}(\mathbf{p})]^T [\mathbf{u}^* - \mathbf{u}(\mathbf{p})] \qquad (1)$$

where $\mathbf{u}^*$ are the measured values and $\mathbf{u}(\mathbf{p})$ are the values calculated using parameters $\mathbf{p}$ for a given model.

Among the various algorithms that might be used to minimize the objective functions, the Gauss-Newton algorithm was chosen, leading to the iteration process described as

$$\mathbf{p}_{k+1} = \mathbf{p}_k + \Delta\mathbf{p}_k \qquad (2)$$

where $\mathbf{p}_k$ is the parameter value vector and $\Delta\mathbf{p}_k$ is the corresponding increment, calculated in iteration *k*.

The iteration expression for the calculation of $\Delta\mathbf{p}_k$ in the minimum square criterion is

$$\Delta\mathbf{p}_k = \left(\mathbf{A}_k{}^T \mathbf{A}_k\right)^{-1} \mathbf{A}_k{}^T \Delta\mathbf{u}_k \qquad (3)$$

where

$$\Delta\mathbf{u}_k = \mathbf{u}^* - \mathbf{u}_k \qquad (4)$$

is the vector defined by the difference between the measured displacements vector $(\mathbf{u}^*)$ and the calculated displacement vector in each iteration $(\mathbf{u}_k)$ and

$$\mathbf{A} = \frac{\partial \mathbf{u}(\mathbf{p})}{\partial \mathbf{p}} \qquad (5)$$

labeled the sensitivity matrix, is a $m \times n$ matrix where m is the number of measured values and n is the number of parameters.

A detailed description of these concepts and of minimization algorithms can be found in [4] and [5].

The determination of an explicit expression for $\mathbf{A}$ demands an also explicit equilibrium equation. This paper uses the equilibrium equation that corresponds to the linear elastic model given by the finite element method, which is

$$\mathbf{K}\,\mathbf{u} = \mathbf{R} \tag{6}$$

where $\mathbf{K}$ is the structure stiffness matrix, $\mathbf{u}$ is the nodal displacement vector and $\mathbf{R}$ is the nodal force vector.

The differentiation of (6) with respect to the parameters yields

$$\frac{\partial}{\partial \mathbf{p}}\left(\mathbf{K}\,\mathbf{u}\right) = \frac{\partial \mathbf{R}}{\partial \mathbf{p}} \quad \Rightarrow \quad \frac{\partial \mathbf{K}}{\partial \mathbf{p}}\mathbf{u} + \mathbf{K}\frac{\partial \mathbf{u}}{\partial \mathbf{p}} = \frac{\partial \mathbf{R}}{\partial \mathbf{p}}$$

from which follows

$$\mathbf{A} = \frac{\partial \mathbf{u}}{\partial \mathbf{p}} = \mathbf{K}^{-1}\left(\frac{\partial \mathbf{R}}{\partial \mathbf{p}} - \frac{\partial \mathbf{K}}{\partial \mathbf{p}}\mathbf{u}\right) \tag{7}$$

which is the sensitivity matrix $\mathbf{A}$ for a finite element method formulation in linear elastic conditions.

An algorithm was developed to identify the parameters to be determined using such concepts. Displacements are calculated through the finite element method using the Adina® software, which places elements of the stiffness matrix and the nodal displacement vector in a simple text file. The iterations were calculated through a Matlab® routine. Figure 2 shows all the steps of the identification method.

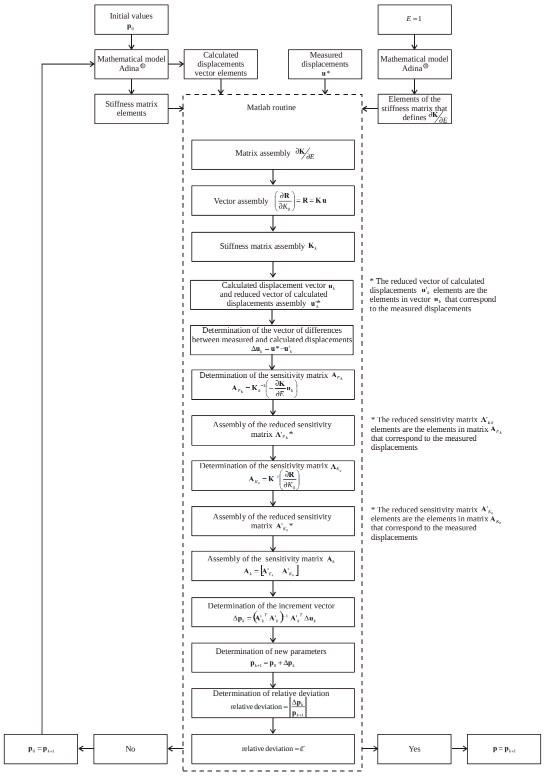

Figure 2. Parameter identification procedure

THE ONE-DIMENSIONAL MODEL

The one-dimensional model, used in the previous works [1], [2] and [3] to identify the reaction of the rockfill dam on the concrete face, considers the concrete slab as being a linear elastic double cantilever beam with variable height. This model is referred to as the beam model. This beam was analyzed by the matrix method of structural analysis. The displacements and the rotations at the ends of the beam were considered to be known support displacements. The hypothesis of linear external load on the elements was adopted.

Figure 3 illustrates the beam discretized by eight elements, in which q_1, q_2, q_3, q_4, q_5, q_6, and q_7 are the load parameters to be identified. They stand for the difference between the known hydrostatic pressure and the rockfill reaction (to be determined). Values of the differences at the left and right ends were defined by preliminary studies, which included assessments based on previous design experience.

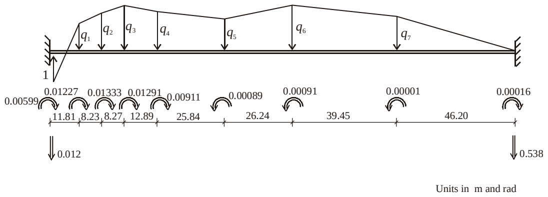

Figure 3. The one-dimensional model and measured rotations

Figure 4 shows the hydrostatic pressure and the obtained rockfill reaction. By applying that load to the double cantilever beam, displacements, rotations and bending moments are obtained.

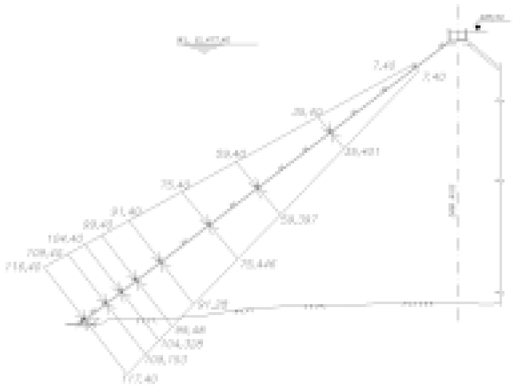

Figure 4. Hydrostatic pressure and obtained reactions

THE TWO-DIMENSIONAL MODEL

The adopted two-dimensional model is discretized by the finite element method. Two different element types are used in this model: the plane strain element and the Bernoulli-Euler beam element. The plane strain element is used to discretize the rockfill and the foundation regions. The dam foundation is characterized by being composed of only one material and the rockfill is divided into seven zones, formed by different materials. The bar element is employed to represent the concrete slab, whose thickness is constant inside the element, but variable in its length. All materials are assumed to be homogeneous, isotropic, and linearly elastic.

Figure 5 identifies the different material regions and Table 1 presents the values of their physical characteristics.

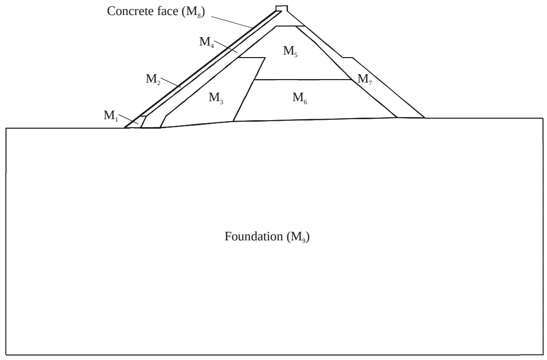

Figure 5. Different material regions

Table 1. Design physical parameters

Material	Young's modulus E (MPa)	Poisson coefficient ν
M_1	140	0.15
M_2	100	0.15
M_3	50	0.15
M_4	80	0.15
M_5	50	0.15
M_6	40	0.15
M_7	80	0.15
M_8	30603	0.2
M_9	30000	0.3

The displacement boundary conditions are defined through the foundation boundary lines, as shown in Figure 6.

The hydrostatic pressure on the concrete face is considered to be the only action on the dam. Due to consolidation injection carried out on the foundation, percolation forces throughout the dam are neglected.

The hydrostatic pressure on a waterproof barrier in the upstream foundation was not considered for simplification. That hypothesis was validated after a series of studies, which verified the low significance of that action on the concrete face.

One holds the deformed configuration due to the gravity action on the foundation and on the rockfill to be the reference configuration. Therefore, gravity forces are not applied to the structure. The constructive approach for the dam practically demands this hypothesis.

Figure 6 illustrates the finite element mesh, the boundary conditions and the applied load used in the model. That model presents 1.599 plane strain elements and 215 bar elements with a total of 3.472 degrees of freedom.

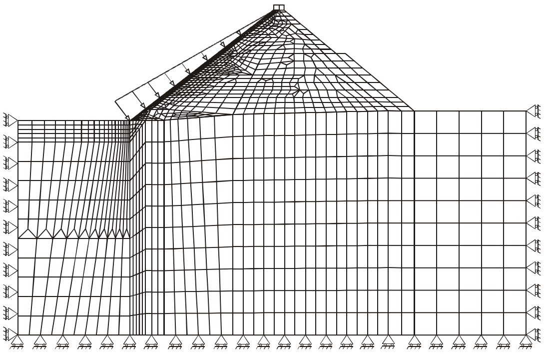

Figure 6. Finite element mesh for the two-dimensional model

Measured rotations are employed to identify E_2, the material M_2 Young's modulus, by application of the iterative procedure presented in Figure 2.

One remarks, in this case, that the nodal force vector $\mathbf{R}$ is independent from the parameter to be identified. Therefore, expression (7) for the sensitivity matrix is reduced to:

$$\mathbf{A} = \frac{\partial \mathbf{u}}{\partial E_2} = \mathbf{K}^{-1}\left(-\frac{\partial \mathbf{K}}{\partial E_2}\mathbf{u} \right) \tag{8}$$

where $\mathbf{K}$ is the structure stiffness matrix and $\mathbf{u}$ is the calculated nodal displacement vector, in correspondence to the measured rotations, in each iteration. One notes that the differentiation of $\mathbf{K}$ with respect to E_2 presents non-zero elements only for those corresponding to the M_2 region, which are conveniently evaluated with $E_2 = 1$.

Under the previous conditions, applying the procedure described in Figure 2 and considering $E_2 = 80.00\text{MPa}$ as the initial estimation value, after seven iterations and with a relative error of 0.91%, the estimated value $E_2 = 71.89\text{MPa}$ is obtained.

RESULTS

With modulus of elasticity $E_2 = 71.89\text{MPa}$, the structural analysis of the two-dimensional model is performed, and displacements, rotations and bending moments on the slab and displacement, stress and strain fields on the rockfill and on the foundation are obtained.

Figure 7 presents the measured rotations, the rotations obtained with the one-dimensional and two-dimensional models and the rotations obtained from the measured rotations and from the ones calculated with the two-dimensional model, both adjusted by polynomials of the sixth order. With the exception of the last one, the different curves present excellent adherence of results. In the initial extension of the concrete slab, all the curves show good adherence.

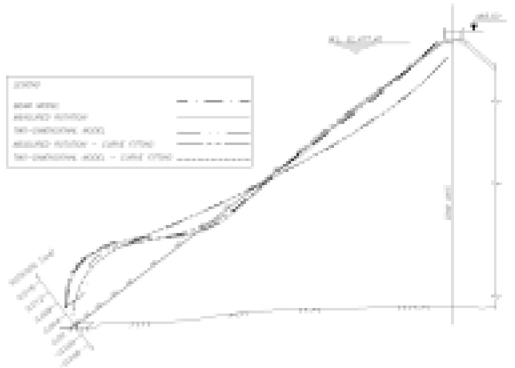

Figure 7. Measured and calculated rotations

Figure 8 presents the slab deformations, characterized by transverse displacements of the slab axis, obtained with the one-dimensional model, with direct integration of the rotations using the tangent method [2] and with the two-dimensional model. The different curves present good adherence of results, mostly in the initial extension of the concrete slab.

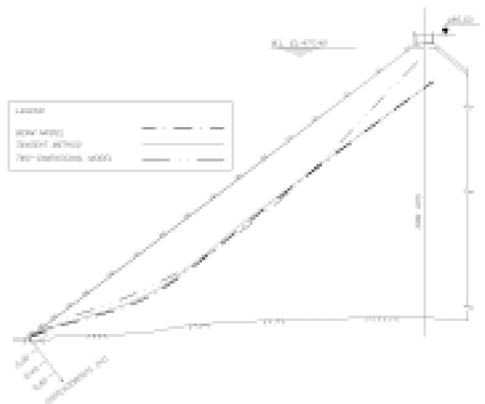

Figure 8. Deformed configurations

Figure 9 shows bending moment diagrams obtained with the one-dimensional model and other two diagrams directly derived, using $M = -EI\theta'$, from adjusted rotation functions of measured rotations and calculated rotations with the two-dimensional model. The curves present a reasonable adherence.

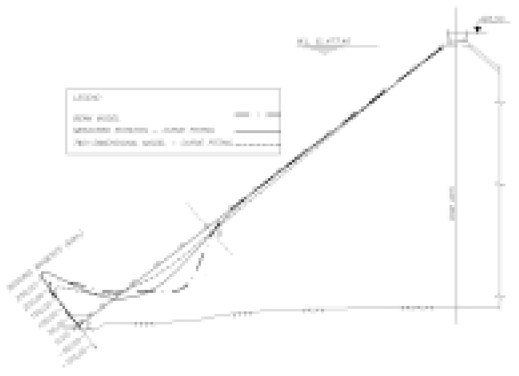

Figure 9. Obtained bending moment diagrams

In order to design the concrete slab of this dam, the significant values of bending moments are the ones corresponding to initial zone AB. For this reason, Figure 10 amplifies and restricts the diagrams of Figure 9 to zone AB. With those results, it is possible to calculate the necessary steel reinforcement and compare it with the recommended by the empirical criteria.

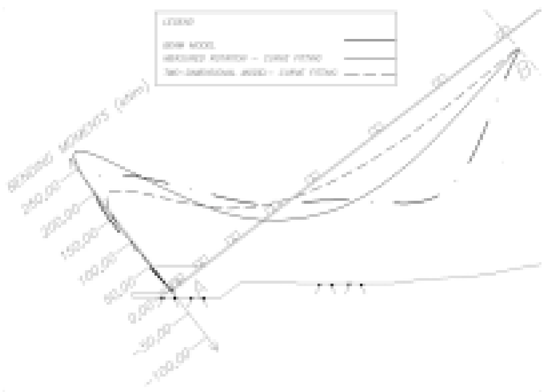

Figure 10. Bending moment diagrams restricted to zone AB

In order to select the finite element mesh to be applied in the back analysis, several studies were carried out using different meshes. Meshes with different refinements presented very good adherence for stress, strain and displacement fields, but were not locally satisfactory, however, for bending moments at the slab's zone AB. This result can be observed in Figure 11, where bending moment diagrams are presented for the back analyzed mesh (Mesh 1) and for a more refined one (Mesh 2), with 5.749 degrees of freedom. One may see some peak at the cross section, which defines the support transition between the rockfill regions M_1 and M_2.

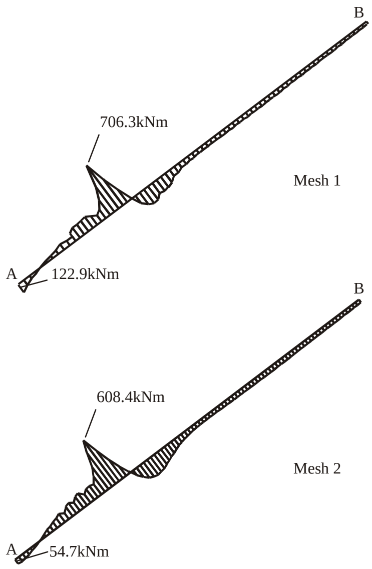

Figure 11. Bending moment diagrams on AB for different meshes

For better comprehension of the peak occurrence, several parametric studies were carried out, considering the same Young's modulus for M_1 and M_2. Under this new condition, Figure 12 presents the bending moment diagrams for different values of the Young's modulus, namely 80, 100 and 140 MPa. All the other regions were considered with the previous values of the modulus of elasticity. The change in the diagrams is noticeable and suggestive with the elimination of the peaks and with significant reduction of the extreme moments.

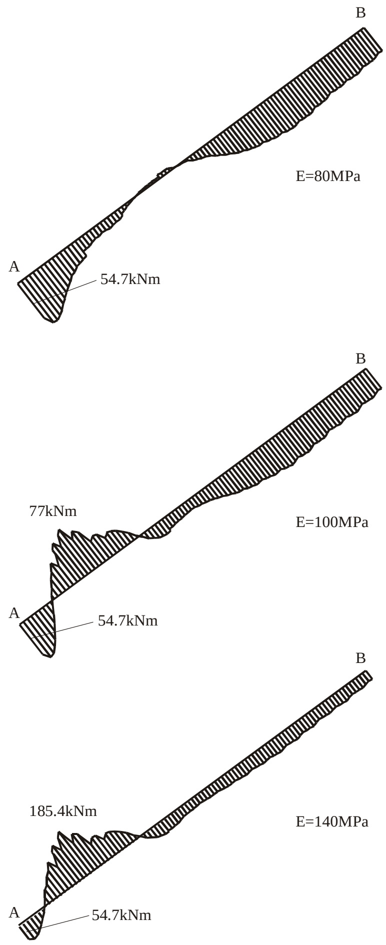

Figura 12. Bending moment diagrams for different values of $E_1 = E_2$

CONCLUDING REMARKS

For back analysis with the objective of obtaining bending moment diagrams in the concrete slab, one considers that the one-dimensional model, although it is a simpler model, presents more satisfactory results than the two-dimensional model. This happens because the one-dimensional model captures with more precision the concrete slab behavior, taking into account the interaction with the rockfill and with the dam foundation. However, this one-dimensional model should not be applied to the straight structural analysis of the concrete slab, due to the impossibility of defining the rockfill reaction on the slab.

For structural analysis, the two-dimensional model is more effective. It provides a better simulation of the rockfill and foundation interaction with the concrete slab. Additionally, it allows higher quality parametric studies, which will permit the structural design of the concrete slab with a better understanding of its behavior. The consideration of the foundation in the two-dimensional model is indispensable to obtain more realistic results.

Smoothing curves, correspondent to displacements and rotations obtained with the two-dimensional model, improve the quality of the results. Extremely refined meshes are avoided close to the slab and some unreal results are eliminated in the shear force diagrams corresponding to the obtained bending moment diagrams.

Both one-dimensional models that include elastic support due to rockfill and foundation effect and non-linear physical behavior of the concrete and two-dimensional models that include the elastic-plastic behavior of the rockfill are being studied for analyses or back analyses, in order to improve the understanding of the behavior of concrete slabs in concrete faced rockfill dams.

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